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Towards Mathematical Literacy in the 21st Century: Perspectives from Indonesia

Wanty Widjaja
<*wanty.widjaja@nie.edu.sg*>
National Institute of Education, Nanyang Technological University, Singapore

Abstract

The notion of mathematical literacy advocated by PISA (OECD, 2006) offers a broader conception for assessing mathematical competences and processes with the main focus on the relevant use of mathematics in life. This notion of mathematical literacy is closely connected to the notion of mathematical modelling whereby mathematics is put to solving real world problems. Indonesia has participated as a partner country in PISA since 2000. The PISA trends in mathematics from 2003 to 2009 revealed unsatisfactory mathematical literacy among 15-year-old students from Indonesia who lagged behind the average of OECD countries. In this paper, exemplary cases will be discussed to examine and to promote mathematical literacy at teacher education level. Lesson ideas and instruments were adapted from PISA released items 2006. The potential of such tasks will be discussed based on case studies of implementing these instruments with samples of pre-service teachers in Yogyakarta.

Key words: Mathematical Modelling, Mathematical Literacy, Indonesia, Mathematical Tasks

Introduction

The notion of mathematical literacy advocated by the Programme for International Student Assessment (PISA) has gained wide acceptance globally. Mathematical literacy goes beyond curricular mathematics and covers a broader conception of what constitutes mathematics. The main focus of PISA assessment is on measuring the potential of 15-year-old students in activating their mathematical knowledge and competencies to solve problems set in real-world situations. PISA‘s (OECD, 2006) definition of mathematical literacy captures this:

> Mathematical literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements, and to engage in mathematics in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen. (OECD, 2006, p. 72).

Indonesia has participated as a partner country in PISA since the start in 2000. The trend from PISA results in mathematics from 2000 to 2009 consistently revealed poor performance. Indonesia is ranked among the lowest performing countries that performed below the OECD average (Table 1). Figure 1 presents changes in some of countries performance from PISA 2003 to 2009.The comparison between 2003 and 2009 results showed that Indonesian 15-year-olds improved their performance by 11 score points. However, a worrying note from the 2009 PISA results was that almost 80% of the Indonesian sample performed below the baseline of level 2 of mathematical literacy. At Level 2, students are expected to show ability to use basic algorithms, formulas, procedures or conventions and use direct reasoning and interpretations of the results (Table 2). The fact that majority of Indonesian students performed below the baseline shows a serious problem with maintaining basic skills in mathematics. Clearly there is a strong impetus to address this problem by improving the quality of mathematics teaching and learning so that more students are mathematically literate.

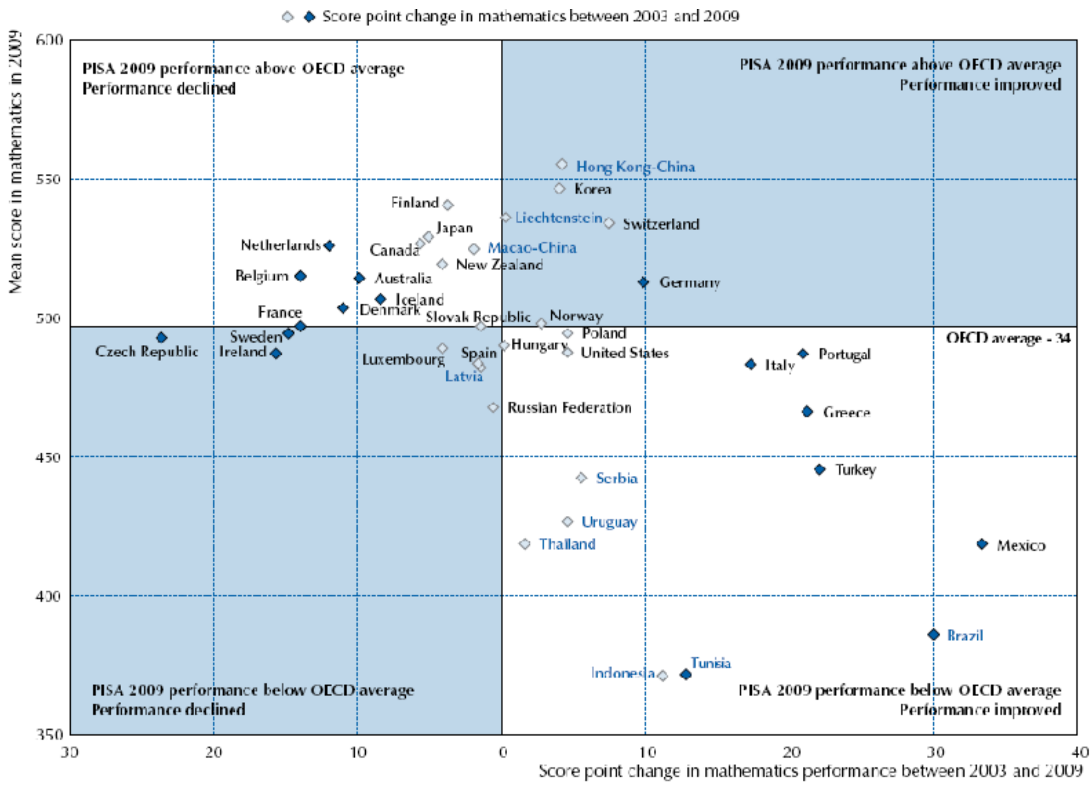

Figure 1. Change in mathematics performance from 2003 to 2009 (OECD, 2010, vol. 5, p. 61)

Table 1
Mean scores of mathematical literary of Indonesian samples in comparison to OECD average [From OECD 2000, Table 3.3., p. 287; OECD 2003, Table 2.5c, p. 356 ; OECD, 2010, Table V.3.1., p. 156]

Country	**Mathematics literacy mean scores (S.E.)**			
	2000	**2003**	**2006**	**2009**
Indonesia	367 (4.5)	360 (3.9)	391 (5.6)	371 (3.7)
OECD average	**500 (0.6)**	**500 (0.6)**	**497 (0.6)**	**499 (0.6)**

Concern over a lack of mathematical literacy among Indonesian students has prompted initiatives to raise more awareness on mathematical literacy. In primary school levels, a reform movement was aimed at placing more emphasis on teaching mathematics connected to real life with PMRI (Sembiring, Hoogland & Dolk, 2010).

Professional development sessions for secondary school teachers have started a few years ago to carry forward the realistic approach of teaching mathematics. Last year, the national council of research and development in Jakarta, 'Balitbang', offered a grant to construct test items for secondary schools similar to PISA items using Indonesian contexts. Some of the items are available from http://pisaindonesia.wordpress.com/aktivitas-pisa-indonesia.

Presently, mathematical literacy contests for secondary school students are being held concurrently in 7 cities in Indonesia to improve mathematical literacy of secondary school students. In addition to such events, a sustainable program at teacher education level is needed to build capacity of future teachers in planning and carrying out lessons that support the development of mathematical literacy.

Mathematical Literacy and Mathematical Modelling

The notion of mathematical literacy is closely connected to the notion of mathematical modelling (Kaiser & Willander, 2005; de Lange, 2006; Stacey, 2009). Mathematical modelling involves cyclical processes which start with a problem situated in a 'real-world' context which is translated and formulated as a mathematical problem. The process of formulating mathematical problems from real-world problems involves simplifying the real-world situations by making assumptions in order to derive mathematical solutions. This process is often referred to as '*mathematisation'* process (de Lange, 2006). The cycle of mathematical modelling ends with interpretations of mathematical solutions in reference to the real-world situations. Evidently, both mathematical modelling and mathematical literacy place the functionality of mathematics in solving real-life situations at the centre of mathematical learning. The descriptors for the top levels (i.e., level 6) of proficiency in mathematics explicitly pinpoint ability to work with models for complex situations and to generalize and utilize information based on the models (Table 2).

Real-world contexts and situations are integral elements of mathematical literacy. However, there is no conclusive voice as to whether real-world contexts in mathematical problem afford or inhibit students' mathematical learning (Rittle-Johnson & Koedinger, 2005; Feijs & de Lange, 2004; Sembiring, Hadi & Dolk, 2008, Widjaja, Dolk, & Fauzan, 2010). Contexts might present a barrier in solving mathematical problems. Movshovitz-Hadar, Zaslavsky, and Inbar (1987) noted that contextual mathematical problems demand linguistic skills which present a barrier on mathematical performance. Prior studies revealed that contexts might not be activated by students due to a tendency for direct translation from a problem into mathematical formulas (Busse, 2005; van den Heuvel-Panhuizen, 1999). However, real-world contexts carry a lot of potential for learning, and allow for multiple pathways to derive at mathematical solutions. Hence the use of real-world contexts cultivates flexible thinking (English, 2010; Lave & Wenger, 1994). Similarly, Widjaja, Dolk and Fauzan (2010) found that meaningful contexts allowed students to relate with their personal experiences which afforded them to solve problems at different levels of mathematical sophistications.

Table 2
Descriptions of mathematical literary of students at level 2, and 6 [From OECD 2010, p. 130]

Level	Lower score limit	What students can typically do
6	669	At Level 6 students can conceptualise, generalise, and utilise information based on their investigations and modelling of complex problem situations. They can link different information sources and representations and flexibly translate between them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situations.
2	420	At Level 2 students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and literal interpretations of the results.

Mathematical Tasks

Two contextualized tasks adapted from the PISA 2006 released items will be discussed (Appendix A, Appendix B). These tasks were assigned to cohorts of Indonesian pre-service teachers as part of their module on '*Teaching strategy of mathematics in secondary school*'. The goal is to expose pre-service teachers to contextualized mathematical tasks and mathematical modelling processes. This exposure is important as future reference in fostering their students' mathematical literacy. The goal of task one given the condition to choose a maximum of 2 toppings was to find combinations of pizzas. Task one was given as a written quiz to be solved in 20 minutes for a class of Indonesian pre-service teachers (Figure 2).

Task two was adapted as a modelling task to investigate a relationship between a person's leg length and his or her pace length. Task two can be considered an extension of the original item whereby a relationship between pace length and the number of steps per minute was given as a mathematical formula. Task two was assigned as group work (four pre-service teachers) to be completed in two weeks. The choice of method of investigation and location for data collection are left open for groups to decide.

Masalah Pizza

[Picture taken from: http://en.wikipedia.org/wiki/Pizza]

Yummy Pizza Shop ingin mengembangkan usahanya. Toko pizza ini menawarkan tiga ukuran pizza: kecil, sedang dan besar.

Pelanggan dapat memilih ***8 topping berbeda***: keju, jamur, nanas, sosis, seafood, ayam, sapi dan sayuran.

Setiap pizza disajikan dengan saus tomat dan taburan keju.

Harga pizza ditentukan oleh ukuran dan banyaknya topping yang dipesan.

Ari ingin memesan pizza ukurang sedang untuk makan malamnya. Karena keterbatasan anggaran, dia hanya dapat memilih ***maksimum 2 topping berbeda***. Berapa banyak variasi pizza yang dapat Ari pilih? Jelaskan jawabanmu selengkap mungkin.

Tuliskan dalam lembar terpisah yang disediakan.

[Problem adapted from: Sáenz, C. (2009). The role of contextual, conceptual and procedural knowledge in activating mathematical competencies (PISA). *Educational Studies in Mathematics, 71*(2), 123–143.]

Figure 2. Task 1 is adapted from PISA 2006 released item and Sáenz (2009) as a written quiz item

Hubungan antara panjang kaki dan panjang langkah

Apakah ada hubungan antara panjang kaki dan panjang langkah seseorang?

1. Tentukan faktor dan variabel dalam masalah ini
2. Kumpulkan data untuk membantu kalian menemukan model matematika. Catat data yang dikumpulkan
3. Apa model yang dapat menjelaskan hubungan antara panjang kaki dan panjang langkah? (Coba cari apakah ada model yang sudah baku).
4. Jelaskan metode kalian dalam menemukan hubungan ini.
5. Setelah menemukan penyelesaian, interpretasikan model yang kalian temukan.
6. Selidiki kembali asumsi yang kalian buat di awal dan berikan masukan untuk perbaikan model kalian.

*Figure 3.*Task 2: A modelling task to investigate the relationships between pace and leg length

Findings

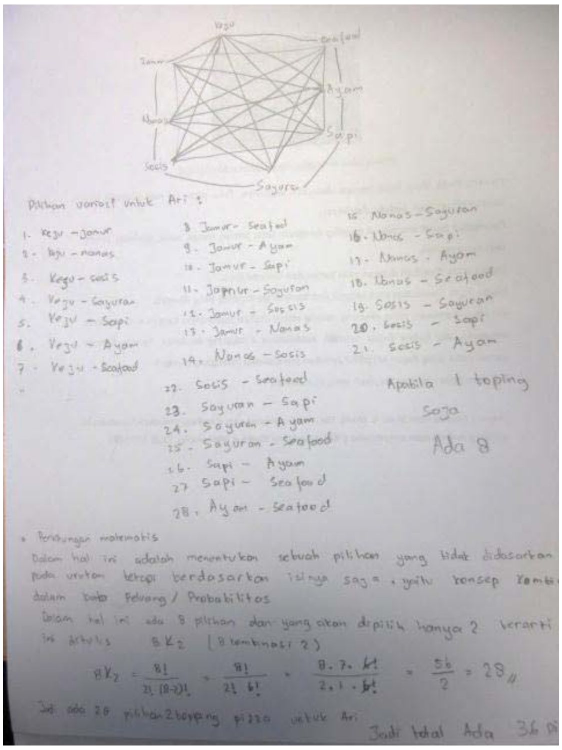

Figure 4a

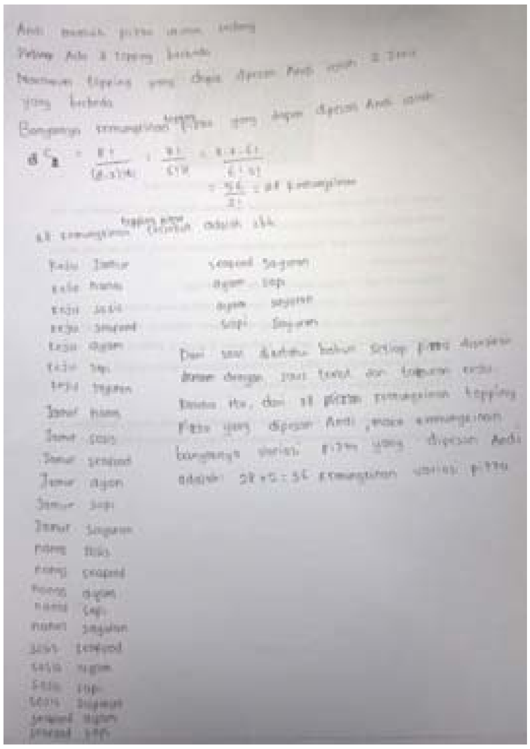

Figure 4b

Figure 4. Samples of responses to Task 1

Pre-service teachers' responses to task one showed that pre-service teachers offered different interpretations for the condition of "*maximum of two different toppings*" as illustrated in a few samples given in Figure 4. A variety of strategies were displayed, e.g., make a list, create a diagram, and use a formula. Both solutions displayed knowledge of relevant formula to solve the problem as well as correct listing of combinations of pizza with two toppings. However, the solution in Figure 4b might suggest that this pre-service teacher could not interpret her solution to the real-world context. The most common incorrect interpretation was disregarding the possibility of having only 1 topping. In this case, only 28 combinations were identified. Some pre-service teachers applied the formulae for finding combinations without reference to the given contexts. For instance, one pre-service teacher found 30 combinations by adding $C^{8}_{2}= \frac{8!}{2!(8-2)!} = \frac{8!}{2!(6)!} = \frac{8\times 7}{2} = 28$ and $C^{2}_{1}= \frac{2!}{1!(2-1)!} = \frac{2!}{1!(1)!} = 2$. This is an example of what Busse (2005) labelled as 'mathematically bound'.

Figure 5. Investigations of relationships between pace and leg length

Implementations of task two with groups of pre-service teachers revealed the potential and challenges faced by pre-service teachers. As illustrated in Figure 5, different ways of investigating the relationships between leg length and pace length were observed. One group decided to collect data by measuring the footsteps of people who walked on the beach. Another group chose to collect data on campus but required volunteers (classmates) to step their feet on paints and walk along the white cloth to obtain a more accurate measurements of pace lengths. The pace lengths were calculated after taking average of four footsteps. Both groups noticed that there were variations among data and it was not a straightforward linear relation based on plotting of the points. A person's mood when walking (e.g., no hurry or in hurry), and locations (e.g., beach or campus) were offered as reasons for non-uniform pace lengths. An assumption such as constant pace length was not made but an average of pace lengths was taken instead. Using the line of best fit, a linear model to explain the relationship between leg length and pace length was derived. Different linear models were offered, $y = 0.526\,x + 12.86$ by the group who collected data on the beach and $y = 0.641\,x - 7.138$, by the group who collected data on campus, with x represents leg length and y represents pace length.

Conclusions

Two tasks from PISA 2006 items were adapted to be use with Indonesian pre-service teachers. The initial findings suggested that contextualized tasks provide opportunities for various strategies. Such tasks allow pre-service teachers to experience the potentials power of mathematics in real-world contexts. Introducing pre-service teachers with contextual tasks and mathematical modelling as part of their training is expected to build capacity of the future teachers to in planning and carrying out lessons that support the development of mathematical literacy. A commitment to place more emphasis on learning processes which present mathematical problems in real-world settings as part of teacher training program is strongly needed. Providing pre-service teachers with such learning experience during their training will better equipped them to make use of their mathematical knowledge and skills in their life.

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Appendix A: PISA 2006 released item (OECD, 2006, p. 71)

M510: Choices

Question 1: CHOICES *M510Q01*

In a pizza restaurant, you can get a basic pizza with two toppings: cheese and tomato. You can also make up your own pizza with **extra** toppings. You can choose from four different extra toppings: olives, ham, mushrooms and salami.

Ross wants to order a pizza with two different **extra** toppings.

How many different combinations can Ross choose from?

Answer: ....combinations.

Appendix B: PISA 2006 released item (OECD, 2006, p. 8)

MATHEMATICS EXAMPLE 26: WALKING

The picture shows the footprints of a man walking. The pacelength P is the distance between the rear of two consecutive footprints.

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between n and P where,

n = number of steps per minute, and

P = pace length in metres.

Question 1: WALKING

Bernard knows his pacelength is 0.80 metres. The formula applies to Bernard's walking.

Calculate Bernard's walking speed in metres per minute and in kilometres per hour. Show your working out.

Curriculum Review in Singapore

The following commentary on the direction and substance of the curriculum review process currently happening in Singapore was included because of its interest to readers from other countries. However, the views expressed here are those of the author, a respected and experienced Asian mathematics educator, and do not necessarily reflect those of the editors or the international advisory panel.

What Might Happen to School Mathematics in 2013?

Lee Peng Yee
National Institute of Education Singapore

The Year 2007

What was new in the O level mathematics syllabus 2007? Perhaps the change was mainly in the teaching approach and not so much in the content knowledge. Among the changes, we put more emphasis on the process and we made an attempt to put mathematics in context.

In fact, there is nothing new about the process and the context. The Chinese did it over 2000 years ago. Look at the classics "Nine Chapters of Arithmetic" and many other classics in mathematics from ancient China. These books normally contained a collection of problems. The problems in these books were always presented in context. The problems were always solved by means of a process. Naturally, there was no formula. If the process itself produced an answer, there was no need for a formula. We have re-discover the approach in modern times. In fact, there is nothing new about it.

There is also nothing new about the process and the context as far as Singapore is concerned. Years ago, more precisely 50 years ago, Euclidean geometry and Newton's mechanics were part of school mathematics at the time. In Euclidean geometry, we prove theorems. To prove a theorem, we have to go through the process. In mechanics, we construct models. To construct a model, we have to put it in context. So there is nothing new about it.

We lost Euclidean geometry gradually over the years starting from the days of the Math Reforms in the 60s in the west and in the 70s in Singapore. As a consequence of the Math Reforms, mathematics became pure mathematics. Gradually, mechanics was replaced by statistics. Henceforth we lost two rich, indeed very rich, areas for learning mathematics and for setting exam questions. We do not know what we have until we have lost it. Now we are trying very hard to recover what we have lost. In other words, we want to teach mathematics with emphasis on the process and pose mathematical problems in real-life or pseudo real-life contexts. In fact, we had it all along in Euclidean geometry and mechanics and then we lost it

In education, there are very few new ideas. People simply re-cycle old ideas and give them new names.

What How When and Why

Let us discuss the what, how, when and why of this so-called new approach to mathematics teaching. In more words, we ask the following questions:

What is it?
How do we do it?
When and where do we do it?
Why do we want to do it?
We shall elaborate in what follows.

What is it?

We want to teach knowledge and we also want to teach the use of knowledge. We want our students to be able to answer the problems in TIMSS and also the problems in PISA. It is said that TIMSS tests the content of mathematics, whereas PISA is more on applications. In other words, we want our students to learn mathematics and also to learn how to apply mathematics.

Put it in practice, we want to set open problems and expect our students to be able to answer them. The key word here is open. To solve such problems, students have to think differently and not rely only on recall.

How do we do it?

We do it by asking open questions. If modelling provides a good way to pose open problems, then use it. Though called by different names such as performance tasks etc, they serve the same purpose as modelling. Note that problems in modelling are always posed in context.

We often use rubrics to mark performance tasks. There should be a distinction between rubrics used for research and rubrics used for classroom assessment. We may not want to go for the full rubrics. An abridged version is more than enough.

When and where do we do it?

The usual comment is that we have no time. We are not expected to do modelling tasks every day. Maybe have it at least once a year or at most once a term, assuming one year has four terms. Asking questions is a way of life, a habit, and an art. Suppose we say the area of a quadrilateral of sides a, b , c, d is $\left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$. Ask not whether the formula is good or bad. Ask how good it is.

Why do we want to do it?

We teach mathematics and we should also teach mathematics for understanding. For understanding, we must look at the process. As I believe, this is known. However we still need formulas and algebra. It is often through formulas that we make mathematics simple and make it easy to apply. Do not condemn formulas. We need both processes and formulas.

The Year 2013

What is new in the O level mathematics syllabus 2013? There is at least one new element in the syllabus that is learning experiences. It is called knowledge requirements in the Swedish mathematics syllabus (2011). Roughly speaking, we make explicit what we want our students to experience during the learning process. For example, when we teach quadratic equations, we want our students to see that the graph of a quadratic function is nothing but a projectile. We can call it an experience and we make it explicit. Further, we can use quadratic functions to find the maximum or minimum point. This is an application. It is another experience, and we make it explicit. In a way, this is a natural consequence of putting emphasis on the process. Now we move one step further to spell out the specific experiences to be emphasized in the learning process.

We are not the only nation introducing this idea in the syllabus. Sweden is doing it. So are the United States in Common Core State Standards (CCSS 2010) and Chile, South America, in Content and Pedagogical Standards for Secondary School Teacher Education in Mathematics (Draft 2011). There is no national mathematics syllabus in the United States. CCSS is the nearest to it. Both the United States and Chile did not call it by a name. But they made it explicit in their syllabuses (standards). However there is one difference. In the 70s, we imported the Math Reforms from the west. This time we did it independently. So were other countries, at least three of them, other than Singapore. As I said above, this is a natural consequence.

You will find all the details of the syllabus 2013 on the website when announced officially. It is a more extensive document than the syllabus 2007. We want to change, but we cannot change overnight. This is only the beginning. We must do it in steps. Probably, this is the only way that we may succeed. This is not fire-fighting, and it should not be fire-fighting. We are training our students in schools for the work place different from our own. Hence we must do it differently.

In The Classroom

A reform, if it is a true reform, can move only as fast as teachers can move. Suppose we, as teachers, believe in the proposed change. Suppose we want to move forward. What are we supposed to do in the classroom? I have no doubt that there will be training programmes. Indeed, it has already started. What I am saying here is something which I think might happen in 2013. Let me quote an email that I sent to my students. Here is the content of the email.

Why is the answer important?

Suppose you build a house. When completed, the house collapsed. Do you still pay the contractor because the process was correct?

Why is the process important?

You may do the wrong thing and still get the right result. I am not sure you will be lucky again next time. If you did it correctly in the process then I have reason to believe that you do not have to depend on luck.

Why is the presentation important?

If you can say it well, you can get through one door. I mean what you say will reach the person behind the door. If you can write well, you can get through at least three doors. I mean what you write will reach the boss of the boss of your boss. Only good presentation travels.

End of the email: In summary, we may wish to do the following in the classroom.

- Make sure mathematics we teach is correct.
- Make explicit the experiences in our learning process.
- Pay more attention to presentation of mathematics.

By all means, teach for the examination. There is nothing wrong to teach for the examination, but not for the examination alone. The examination will not change in the short term. In times to come, it will change. It will be too late for students to catch up if we do not start changing our teaching now. We must accept the fact that though we may give the same amount to all students, students may not receive the same amount at their end. Since we apply differentiated syllabuses, differentiated teaching, therefore we must also accept differentiated learning.

As I said it elsewhere, we keep changing and changing, we reach a point that we have nowhere to copy from and we have to find our own solution to our unique problem. We, I mean curriculum designers, teacher trainers, and teachers, have to do it together to find a way of moving forward, a way that works for us. For example, we may have to build up jointly the resources to be used in the classroom.

In conclusion, we must teach mathematics differently. We want our students to be able to solve problems beyond the textbooks. Perhaps teaching from the syllabus will no longer be an exception. However we may not want to do that all the time. One important thing to remember is that students must learn how to follow rules first before learning how to break them.

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[Note: Some of the materials above have been given in a talk to Singapore teachers on 08 September 2011]